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SELF-ORGANIZATION AND DYNAMIC CHAOS IN CHEMICAL-TECHNOLOGY AND HEAT-EXCHANGE DEVICES: PROBLEMS AND TASKS

L. P. Kholpanov UDC 577.3; 517.9; 536.1; 66.02:621.1:533:51-74

Nonlinear phenomena in thermophysical and chemical-technology processes are considered in the context of nonlinearity, instability, nonuniqueness, nonstationarity, and irreversibility. The perturbation frequency (phase)-to-amplitude ratio is the governing parameter of the nonlinear interaction of perturbations that are described by the proposed general nonlinear parabolic equation. The nonlinearity of this ratio characterizes turbulence, while its linearity characterizes self-organization. It is shown that chaotic conditions can be self-organized under the action of "white noise," which favors the system getting into the domain of attraction of a stable node. The mechanism of the occurrence of turbulence is substantiated.

Intense heat- and mass-exchange processes in chemical-technology and heat-exchange devices are accompanied by the development of unstable conditions in separate subsystems: laminar conditions of flow of liquid films change to wave and turbulent conditions; Taylor instability develops in bubbling apparatuses; Marangoni instability appears on the liquid-phase surface in mass-exchange apparatuses; convection currents are formed in nonuniformly heated volumes. In most cases, it is impossible to prevent the development of instability, and sometimes it is not expedient, since the resulting instabilities intensify the heat- and mass-exchange processes.

The development of instability leads to the appearance of self-ordered monochromatic, low-mode chaotic, and multimode turbulent conditions [1–3].

The chaotic and turbulent conditions are characterized by considerable spreads in hydrodynamic and heat- and mass-exchange parameters. The probability that the system will attain emergency operating conditions in the case of chaotic instability is higher than in the case of its other types.

The monochromatic conditions combine high intensity of heat- and mass-exchange processes, which is caused by the ordered convective motion of a liquid, and insignificant spreads in heat- and mass-exchange parameters and stability to perturbations. In order to prevent emergency situations in highly efficient processes, it is worthwhile to use self-organizing conditions; for this purpose, one must investigate the conditions of occurrence and the laws of long-term development of unstable regimes, carry out their classification, and find conditions for the change of the chaotic and turbulent conditions to self-organizing ones.

Modern nonlinear dynamics experiences fundamental changes. Unpredictable and chaotic-looking random vibrations appear in nonlinear dynamic systems (described by nonlinear equations with regular (nonrandom) coefficients) that execute vibrations under the action of regular external forces of a periodic and especially a nonperiodic nature. In other words, solutions of these nonlinear equations depend strongly on initial conditions. Another property of chaotic vibrations is that they "forget" about the initial conditions. Chaotic dynamics (or dynamic chaos), which is characteristic of all the nonlinear phenomena of inanimate and

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Institute for Problems of Chemical Physics, Russian Academy of Sciences, Chernogolovka, Moscow Region, Russia; email: kholp@icp.ac.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 4, pp. 5–13, July–August, 2001. Original article submitted September 7, 2000.

animate nature, caused a revolution in modern nonlinear dynamics. Until recently, three types of dynamic motion were known: equilibrium motion, periodic (or the limiting cycle), and quasiperiodic motion. These states of dynamic systems on a phase plane were called attractors, since in damping of motion or in the case of stability loss the system "is attracted" to one of the above-mentioned states. Chaotic vibrations and their nonlinear interactions produced a new type of motion that is related to the state called the "strange" attractor thanks to the works of Ruelle, Takens, and Lorentz.

The appearance of attractors of different form is associated with the nonlinear interaction of evolving perturbations. Several models are available (the Ginzburg–Landau, Swift–Hohenberg, Newell–Whitehead–Segel, and Van der Pol models) that describe bifurcation processes in nonlinear systems. However, when these models are used many problems which are associated with account for various types of nonlinear interactions between perturbations still remain to be solved.

General Nonlinear Parabolic Equation (GNPE). A large class of unstable thermophysical, physicochemical, chemically reacting, electrochemical, physical, biological, and hydrodynamic nonlinear processes are described by the well-known systems of nonlinear partial differential equations

$$\frac{\partial\varphi}{\partial t} + \mathbf{N}(\varphi) + \sum_{i=1}^{2} \mathbf{Q}_i(\varphi)\frac{\partial\varphi}{\partial x_i} + \sum_{ij=1}^{2} \mathbf{M}_{ij}(\varphi)\frac{\partial^2\varphi}{\partial x_i\,\partial x_j} = 0\,, \tag{1}$$

where $\varphi = \|\varphi_1...\varphi_n\|^T$ is the real vector determined in the region $D = \{(t, x_1x_2) |\ t \geq 0,\ -\infty < x_i < \infty\}$, $\mathbf{N} = \|N_1...N_n\|^T$, $\mathbf{Q}$ and $\mathbf{M}$ are $n \times n$ matrices, and $\mathbf{M}_{12} = \mathbf{M}_{21}$. The first two terms in Eq. (1) are well-known equations of kinetics and biophysics, while, with the appropriate concretization of the form of the coefficients, they are the equations of the kinetics of Belousov–Zhabotinskii-type reactions. The first, second, and fourth terms describe the same reactions with diffusion and also heat conduction with nonlinear sources (sinks) of energy. In complete form, equations of the type (1) characterize various hydrodynamic phenomena and can be rearranged to a system of quasilinear differential equations with source (sink) terms that is investigated in [4]. Generally, this system can describe combined processes, for example, convective heat- and mass-exchange, since for the vector $\mathbf{N}$ restrictions are not imposed on the nature of a substance.

Let the system of equations (1) allow the stationary solution $\varphi = \varphi_0$, which in an open system, as a result of an external action or in a random way, loses stability for certain values of the parameters. In system (1), perturbations belonging to a continuous spectral band of wave numbers are excited and grow in the supercritical region.

We assign the perturbed solution of system (1) in the form

$$\varphi = \varphi_0 + \tilde{\varphi}\,. \tag{2}$$

The problem of nonlinear development of perturbations from the continuous spectral band of wave numbers is solved using wave packets [5–10]

$$\tilde{\varphi} = \int_{k_{10}-\Delta k_1}^{k_{10}+\Delta k_1} \int_{k_{20}-\Delta k_2}^{k_{20}+\Delta k_2} F(k_1, k_2) \exp i\,(k_1x_1 + k_2x_2 - \omega t)\, dk_1dk_2 + \text{com. con}\,, \tag{3}$$

where k_{10} and k_{20} are the centers of the wave packet along the x_1 and x_2 axes, respectively, Δk_1 and Δk_2 are the wave-packet widths along the x_1 and x_2 axes, respectively, and $\omega = \omega_r + i\omega_i$ is the complex frequency; com.con are the complex conjugate quantities

Under the assumption that

$$\frac{\Delta k_1}{k_{10}} = o\,(\varepsilon)\,; \quad \frac{\Delta k_2}{k_{20}} = o\,(\varepsilon)\,; \quad \frac{\tilde{\varphi}}{\varphi_0} = o\,(\varepsilon)\,; \quad (\partial\omega_i/\partial k)/\omega_{i=0} = o\,(\varepsilon)\,; \quad \varepsilon << 1\,, \tag{4}$$

the spectrally narrow wave packet will be represented in the form of a quasimonochromatic wave

$$\tilde{\varphi} = \int\limits_{k_{10}-\Delta k_1}^{k_{10}+\Delta k_1} \int\limits_{k_{20}-\Delta k_2}^{k_{20}+\Delta k_2} F\,(k_1, k_2)\,\exp i\,(k_1 x_1 + k_2 x_2 - \omega t)\,dk_1 dk_2 + \text{com. con} = A \exp i\,(k_{10}x_1 + k_{20}x_2 -$$

$$- \omega\,(k_{10}, k_{20})\,t) + \text{com. con} = a\,(\varepsilon t, \varepsilon^2 t, \varepsilon x_1, \varepsilon x_2)\exp i\theta\,(\varepsilon t)\exp i\,(k_{10}x_1 + k_2 x_2) + \text{com. con}\,, \tag{5}$$

where

$$A = \int\limits_{-\Delta k_1}^{\Delta k_1} \int\limits_{-\Delta k_2}^{\Delta k_2} F\,(k_{10} + \delta k_1, k_2 + \delta k_2)\exp i\left(\delta k_1 x_1 + \delta k_2 x_2 - \left(\frac{\partial\omega}{\partial k_1}\right)\delta k_1 t - \left(\frac{\partial\omega}{\partial k_2}\right)\delta k_2 t - \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_1^2}\right)(\delta k_1)^2\,t -\right.$$

$$\left. - \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_2^2}\right)(\delta k_2)^2\,t - \left(\frac{\partial^2\omega}{\partial k_1,\partial k_2}\right)\delta k_1\delta k_2 t\right)\partial\delta k_1\partial\delta k_2 + o\,(\varepsilon^3) =$$

$$= A\,(x_{11}, x_{21}, x_{12}, x_{22}, t_1, t_2) + o\,(\varepsilon^3)\,. \tag{6}$$

From Eqs. (5) and (6) it follows that the sum of m harmonics of belonging to the spectrally narrow wave packet can be represented in the form of quasimonochromatic waves; here the amplitude a and phase θ, as is obvious from Eq. (5), are functions of slow variables (εt and εx):

$$t_0 = t\,;\; t_1 = \varepsilon t\,;\; t_2 = \varepsilon^2 t\,;\; x_1 = x_{10}\,;\; x_2 = x_{20}\,;\; x_{11} = \varepsilon x_1\,;\; x_{21} = \varepsilon x_2\,;\; x_{12} = \varepsilon^2 x_1\,;\; x_{22} = \varepsilon^2 x_2\,. \tag{7}$$

We introduce the expansion

$$\varphi = \varphi_0 + \sum_{j=1}^{\infty}\sum_{l=-\infty}^{\infty} \varepsilon^j A_l^{(j)} \exp il\,(k_1 x_{10} + k_2 x_{20} - \omega_r t_0) + \text{com. con}\,, \tag{8}$$

as well as the following operators which take into account the factor that the processes occur on many scales:

$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial x_{10}} + \varepsilon\frac{\partial}{\partial\eta_1} + \varepsilon^2\frac{\partial}{\partial x_{12}}\,; \quad \frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_{20}} + \varepsilon\frac{\partial}{\partial\eta_2} + \varepsilon^2\frac{\partial}{\partial x_{22}}\,;$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} - \varepsilon\frac{\partial\omega_r}{\partial k_1}\frac{\partial}{\partial\eta_1} - \varepsilon\frac{\partial\omega_r}{\partial k_2}\frac{\partial}{\partial\eta_2} + \varepsilon^2\frac{\partial}{\partial t_2}\,; \tag{9}$$

$$\eta_1 = \varepsilon\left(x_1 - \frac{\partial\omega_r}{\partial k_1}t\right); \quad \eta_2 = \varepsilon\left(x_2 - \frac{\partial\omega_r}{\partial k_2}t\right); \quad \mathbf{A}_l^{(j)} = |A_{l1}^{(j)} \ldots A_{1n}^{(j)}|^{\mathrm{t}}\,, \tag{10}$$

where $\mathbf{A}_{-l}^{(j)}$ is the vector that is complex conjugate to $\mathbf{A}_l^{(j)}$ and η is the wave coordinate.

Thus, the reduction of the system of equations (1) to the equation for the amplitude of a nonlinear perturbation is carried out in a comprehensive manner, i.e., using the wave packets (3), the methods of many scales (9), and modification of the Mandel'shtam method according to which *m* harmonic waves with different wave numbers and frequencies are rearranged to the form of quasimonochromatic waves with nonlinear amplitude and phase depending on the slow variables. This idea was used in transforming the spectrally narrow wave packet (formulas (5) and (6)). Finally, we used transformation (10), which takes into account the group velocity of the envelope wave, which is typical for the actual nonlinear dispersion medium.

The nonlinear system of differential equations in partial derivatives in the εth approximation, obtained after the substitution of expansion (8) into system (1) with allowance for (9) and (10), becomes inconsistent. For its solvability, it is required that the right-hand side of the system obtained would be orthogonal to any solution of a homogeneous conjugate system, as was performed previously [5–7]. This difficulty is resolved by means of another method [10].

With (9) and (10) taken into account, in the system of differential equations obtained after the substitution of (8) into (1) we separate the linear side and denote it by the matrix operator as **L**, while the remaining nonlinear side is also denoted by the matrix operator of the same system as **V**. Then the system of equations obtained can be represented in the form

$$\mathbf{LX} = \mathbf{V}\,, \tag{11}$$

in which

$$\mathbf{X} = \left\| \begin{matrix} x_1 \\ x_2 \\ \ldots\ldots \\ \ldots\ldots \\ x_n \end{matrix} \right\| \; ; \quad \mathbf{V} = \left\| \begin{matrix} V_1 \\ V_2 \\ \ldots\ldots \\ \ldots\ldots \\ V_n \end{matrix} \right\| \; ; \quad \mathbf{L} = \left\| \begin{matrix} L_{11} L_{12} \ldots L_{1n} \\ L_{21}\, L_{22} \ldots L_{2n} \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ L_{n1}\, L_{n2} \ldots L_{nn} \end{matrix} \right\| \,. \tag{12}$$

Having multiplied the left- and right-hand sides of Eq. (11) by the adjoint matrix $\mathbf{L}^*$, we obtain

$$\mathbf{L}^*\mathbf{LX} = \mathbf{L}^*\mathbf{V}\,. \tag{13}$$

The expansion of these matrices in a small parameter ε gives

$$\mathbf{L} = \mathbf{L}_0 + \varepsilon\mathbf{L}_1 + \varepsilon^2\mathbf{L}_2 + \varepsilon^3\mathbf{L}_3\,; \quad \mathbf{X} = \varepsilon\mathbf{X}_1 + \varepsilon^2\mathbf{X}_2 + \varepsilon^3\mathbf{X}_3\,; \quad \mathbf{L}^* = \mathbf{L}_0^* + \varepsilon\mathbf{L}_1^* + \varepsilon^2\mathbf{L}_2^* + \varepsilon^3\mathbf{L}_3^*\,;$$

$$\mathbf{V} = \varepsilon^2\mathbf{V}_2 + \varepsilon^3\mathbf{V}_3\,.$$

We collect the terms for identical ε^k, $k = 0, 1, 2$, and 3:

$$\text{for } \varepsilon^1 :\; \mathbf{L}_0^*\,\mathbf{L}_0\,\mathbf{X}_1 = 0\,;$$

$$\text{for } \varepsilon^2 :\; (\mathbf{L}_0^*\,\mathbf{L}_1 + \mathbf{L}_1^*\,\mathbf{L}_0)\,\mathbf{X}_1 + \mathbf{L}_0^*\,\mathbf{L}_0\,\mathbf{X}_2 = \mathbf{L}_0^*\,\mathbf{V}_2\,;$$

$$\text{for } \varepsilon^3 :\; (\mathbf{L}_0^*\,\mathbf{L}_2 + \mathbf{L}_1^*\,\mathbf{L}_1 + \mathbf{L}_2^*\,\mathbf{L}_0)\,\mathbf{X}_1 + (\mathbf{L}_0^*\,\mathbf{L}_1 + \mathbf{L}_1^*\,\mathbf{L}_0)\,\mathbf{X}_2 + \mathbf{L}_0^*\,\mathbf{L}_0\,\mathbf{X}_3 = \mathbf{L}_1^*\,\mathbf{V}_2 + \mathbf{L}_0^*\,\mathbf{V}_3\,; \tag{14}$$

$$\text{for } \varepsilon^4 :\; \ldots\ldots\,.$$

Eliminating successively the secular terms of different approximations in system (14), we separate the secular terms of the third approximation. This procedure makes it possible to obtain the following equation for the amplitude of the envelope wave in the third approximation:

$$\frac{\partial A_0}{\partial t_2}+\frac{\partial \omega_r}{\partial k_1}\frac{\partial A_0}{\partial x_{12}}+\frac{\partial \omega_r}{\partial k_2}\frac{\partial A_0}{\partial x_{22}}+\frac{i}{\varepsilon}\frac{\partial \omega_i}{\partial k_1}\frac{\partial A_0}{\partial \eta_1}+\frac{i}{\varepsilon}\frac{\partial \omega_i}{\partial k_2}\frac{\partial A_0}{\partial \eta_2}-$$

$$-\frac{i}{2}\left(\frac{\partial^2 \omega_r}{\partial k_1^2}+i\frac{\partial^2 \omega_i}{\partial k_1^2}\right)\frac{\partial^2 A_0}{\partial \eta_1^2}-\frac{i}{2}\left(\frac{\partial^2 \omega_r}{\partial k_2^2}+i\frac{\partial^2 \omega_i}{\partial k_2^2}\right)\frac{\partial^2 A_0}{\partial \eta_2^2}-$$

$$-i\left(\frac{\partial^2 \omega_r}{\partial k_1 \partial k_2}+i\frac{\partial^2 \omega_i}{\partial k_1 \partial k_2}\right)\frac{\partial^2 A_0}{\partial \eta_1 \partial \eta_2}+(\beta_1+i\beta_2)\,|A_0|^2 A_0=0\,, \tag{15}$$

where β_1 and β_2 are the Landau constants; here β_1 characterizes the nonlinear damping of perturbations and β_2 characterizes the nonlinear dispersion. These constants can be obtained from the previous approximation.

Substituting into (15) the wave amplitude A_0 in the form

$$A_0=A_+\exp i\left[\delta k_s x_s+\left(\frac{\partial \omega_r}{\partial k_s}\,\delta k_s+\frac{1}{2}\frac{\partial^2 \omega_r}{\partial k_s \partial k_t}\,\delta k_s \delta k_t\right)t\right],$$

where the double subscripts in $\delta k_s x_s$ indicate the summation, and going from the scaling variables t_2, x_{12}, and x_{22} to the variables t_2, $\eta_{12} = x_{12}-\frac{\partial \omega_r}{\partial k_1}t_2$, and $\eta_{22} = x_{22}-\frac{\partial \omega_r}{\partial k_2}t_2$ for the amplitude A_+ of the wave packet, whose center is shifted by δk_1 and δk_2 from the neutral-stability curve, we obtain a general two-dimensional nonlinear parabolic equation in the form

$$\frac{\partial A_+}{\partial t_2}+\frac{i}{\varepsilon}\left(\frac{\partial \omega_i}{\partial k_1}\frac{\partial A_+}{\partial \eta_1}+\frac{\partial \omega_i}{\partial k_2}\frac{\partial A_+}{\partial \eta_2}\right)-\frac{1}{2}\left(\frac{\partial^2 \omega_r}{\partial k_1^2}+i\frac{\partial^2 \omega_i}{\partial k_i^2}\right)\frac{\partial^2 A_+}{\partial \eta_1^2}-$$

$$-\frac{i}{2}\left(\frac{\partial^2 \omega_r}{\partial k_2^2}+i\frac{\partial^2 \omega_i}{\partial k_2^2}\right)\frac{\partial^2 A_+}{\partial \eta_2^2}-i\left(\frac{\partial^2 \omega_r}{\partial k_1 \partial k_2}+i\frac{\partial^2 \omega_{i2}}{\partial k_1 \partial k_2}\right)\frac{\partial^2 A_+}{\partial \eta_1 \partial \eta_2}=$$

$$=\frac{\omega_i}{\varepsilon^2}A_+-(\beta_1+i\beta_2)\,|A_+|^2 A_+\,. \tag{16}$$

Now we write (16) in dimensionless variables:

$$\eta_{10}=\eta_1\sqrt{\left(\frac{2\omega_i}{\varepsilon^2\left|\frac{\partial^2 \omega_i}{\partial k_1^2}\right|}\right)};\quad \eta_{20}=\eta_2\sqrt{\left(\frac{2\omega_i}{\varepsilon^2\left|\frac{\partial^2 \omega_i}{\partial k_2^2}\right|}\right)};\quad \tau=\frac{t_2\omega_1}{\varepsilon^2}\,;\quad A=A_+\sqrt{\frac{\varepsilon^2\beta_i}{\omega_i}}\,; \tag{17}$$

then

$$\frac{\partial A}{\partial \tau} + i\left(\alpha_{31}\frac{\partial A}{\partial \eta_{10}} + \alpha_{32}\frac{\partial A}{\partial \eta_{20}}\right) + \left(\operatorname{sgn}\frac{\partial^2 \omega_i}{\partial k_1^2} - i\alpha_{11}\right)\frac{\partial^2 A}{\partial \eta_{10}^2} +$$

$$+\left(\operatorname{sgn}\frac{\partial^2 \omega_i}{\partial k_2^2} - i\alpha_{12}\right)\frac{\partial^2 A}{\partial \eta_{20}^2} + (\alpha_r - i\alpha_i)\frac{\partial^2 A}{\partial \eta_{10}\partial \eta_{20}} = A - (1 + i\alpha_2)\,|A|^2 A\,, \tag{18}$$

where

$$\alpha_{11} = \frac{\partial^2\omega_r/\partial k_1^2}{\partial^2\omega_i/\partial k_1^2}; \quad \alpha_{12} = \frac{\partial^2\omega_r/\partial k_2^2}{\partial^2\omega_i/\partial k_2^2}; \quad \alpha_2 = \frac{\beta_2}{\beta_1}; \quad \alpha_{31} = \frac{\partial \omega_i}{\partial k_1}\sqrt{\left(\frac{2}{\omega_i\left|\frac{\partial^2\omega_i}{\partial k_1^2}\right|}\right)}; \quad \alpha_{32} = \frac{\partial \omega_i}{\partial k_2}\sqrt{\left(\frac{2}{\omega_i\left|\frac{\partial^2\omega_i}{\partial k_2^2}\right|}\right)};$$

$$\alpha_r = \frac{2\frac{\partial^2\omega_i}{\partial k_1\partial k_2}}{\sqrt{\left(\left|\frac{\partial^2\omega_i}{\partial k_1^2}\right|\left|\frac{\partial^2\omega_i}{\partial k_2^2}\right|\right)}}; \quad \alpha_i = \frac{2\frac{\partial^2\omega_i}{\partial k_1\partial k_2}}{\sqrt{\left(\left|\frac{\partial^2\omega_i}{\partial k_1^2}\right|\left|\frac{\partial^2\omega_i}{\partial k_2^2}\right|\right)}}. \tag{19}$$

Analysis of Numerical Solutions of the General Nonlinear Parabolic Equation. The dimensionless numbers (19) have a simple physical meaning: α_{1j} characterizes the ratio of the dispersion of the group velocity in the jth direction to the dispersion of the increment; α_{3j}, the deviation of the center of the wave packet in the jth direction from the harmonic of the maximum increment; α_2 characterizes the nonlinear dependence of the phase (frequency) on the amplitude, i.e., nonlinear dispersion.

Equation (18) describes the evolution of the envelope wave that appeared as a result of the nonlinear interaction of perturbations assigned in the form of a wave packet. As particular cases, Eq. (18) gives the well-known relations from [11, 12].

To characterize the behavior of the dynamic systems, we used the Lyapunov exponents, the Kolmogorov–Sinai entropy, and the Poincaré mapping.

For example, for conservative systems of plasma physics, nonlinear optics, and the hydrodynamics of an ideal liquid with $\omega_i = \beta_i = 0$, Eq. (18) is reduced to the well-known Schrödinger nonlinear parabolic equation [11, 12].

The Lyapunov exponents defined for nonlinear dynamic systems by the method of [13] are the quantitative characteristic of the attractor. For system (18), the Lyapunov exponents were in the space a_n and θ_n:

$$\lambda_n(a_{n0}, \theta_{n0}) = \lim_{\tau\to\infty}\frac{1}{\tau}\ln|u(\tau, a_{n0}, \theta_{n0})|\,,$$

where $u(\tau, a_{n0}, \theta_{n0})$ is the vector of sensitivity functions (tangential flow). Moreover, if all the Lyapunov exponents λ_n are negative, the system is constricted in all directions of a_n and θ_n, and the stable node or stable focus is the attractor in the space a_n and θ_n. The vanishing of one of the Lyapunov exponents in the case of negativeness of all the remaining exponents indicates the presence of a limiting cycle. The equality of n Lyapunov exponents to zero when the remaining exponents are negative corresponds to the existence of an n-dimensional torus. The appearance of positive exponents reveals the occurrence of a stochastic attractor.

In addition, in order to characterize the dynamic chaos, we used the Kolmogorov–Sinai entropy. This metric invariant of dynamic systems was introduced for the first time by Kolmogorov [14] and subsequently

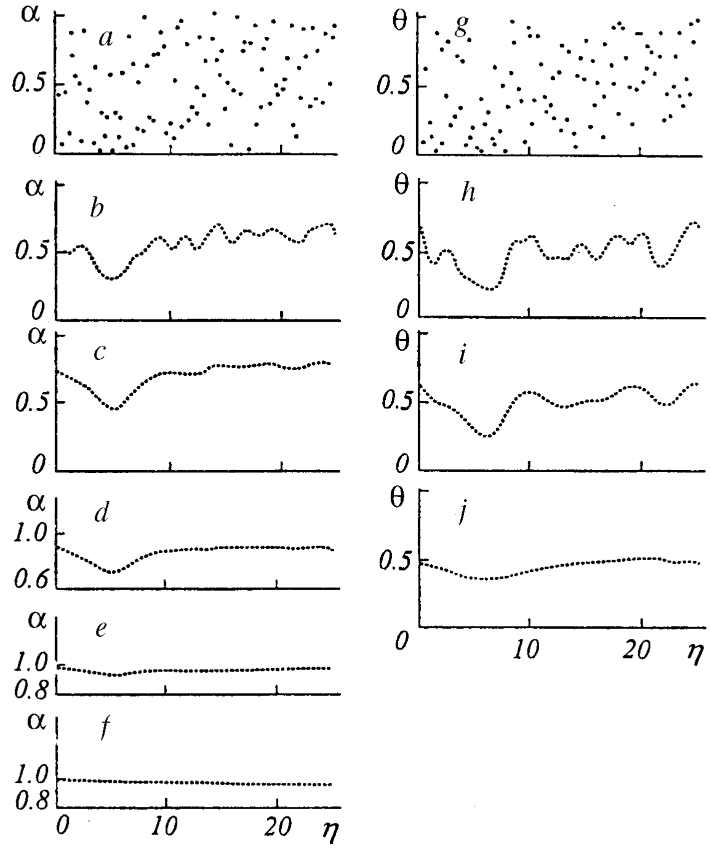


Fig. 1. Self-organization of the random field of perturbations of the amplitude (a–f) and phase (g–j) in the medium without dispersion ($\alpha_{11} = \alpha_{12} = \alpha_2 = 0$) for $\tau = 0$ (a, g), 0.2 (b, h), 0.6 (c, i), 1.2 (d), 2.0 (e), and 4.8 (f, j). Owing to self-organization, the values of α and θ are constant at all points of the space.

was developed by Sinai [15]. The measure of the development of turbulence was the positiveness of the metric invariant of dynamic systems that results in the scattering of exponential phase trajectories.

Finally, to characterize the behavior of dynamic systems we also used the Poincaré mapping.

The complex investigation of these propositions for the quantitative and qualitative characteristic of nonlinear dynamic systems made it possible to track qualitative changes in the trajectories of these systems with change in the parameters (19) in the numerical solution of the general parabolic equation (18), i.e., to investigate bifurcation. Here, in different stages of evolution of the nonlinear interaction of perturbations, certain sequences of bifurcations leading to a strange attractor showed up, namely, bifurcations that cause intermittency, doubling of the Feigenbaum period, production and failure of a three-dimensional torus, and, finally, a sequence that results in a homoclinic contour.

Based on the numerical solution of Eq. (18) [5–10], it is established that all the governing parameters α_{1j}, α_{3j}, and α_2, called by us the criteria, affect the behavior of a perturbation; however, the parameter α_2 is of primary importance in rearrangement of the perturbations that determine the appearance of either self-organization or chaos (turbulence). Account for the influence of α_2 and other criteria in Eq. (19) on the nature of the interaction and development of perturbations allowed us to establish the following laws of occurrence of self-organization and turbulence (chaos) and of the transition between them in hydrodynamic, thermophysical, physical, chemically reacting, and biological systems [9, 10]:

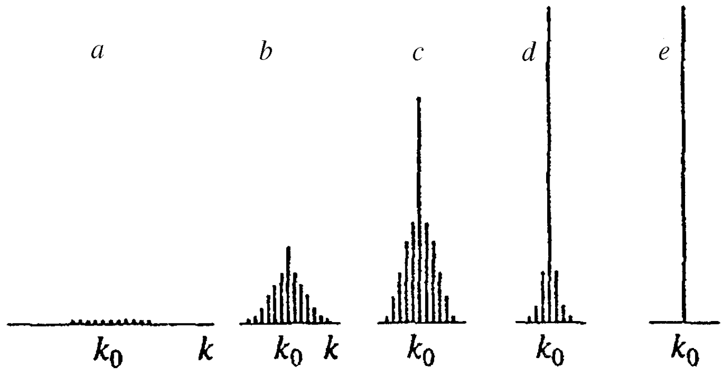


Fig. 2. Contraction of the wave packet in the case of occurrence of self-organization in the medium without dispersion ($\alpha_{11} = \alpha_{12} = \alpha_2 = 0$) for $\tau = 0$ (a), 1.5 (b), 4 (c), 8 (d), and 14 (e).

1. The linear dependence of the phase or frequency on the amplitude of a perturbation is the necessary condition for self-organization. Degenerate cases of this linear dependence are also possible: the slope or the free term, or both simultaneously are equal to zero (Figs. 1 and 2).

2. The basic condition for the occurence of turbulence (chaos) is the nonlinear dependence of the phase for distributed systems or the frequency for nondistributed systems on the perturbation amplitude (Figs. 3 and 4). An increase in the dispersion favors the occurrence of self-organization (Fig. 5).

3. In chaotic and disordered systems, self-organization, coherent structures, and order occur owing to the nonlinear interaction of perturbations, and conversely.

4. The transition from one type of nonlinear interaction to another and the appearance of the structure or phenomenon are accompanied by a change in the energy distribution over the spectrum, i.e., by the contraction (self-organization) (Fig. 2) or expansion (turbulence) (Fig. 4) of energy over the spectrum. An intermediate state is possible.

The indicated laws are obtained from the analysis of the numerical solutions of Eq. (18) and can be used for controlling self-organization and turbulence (chaos).

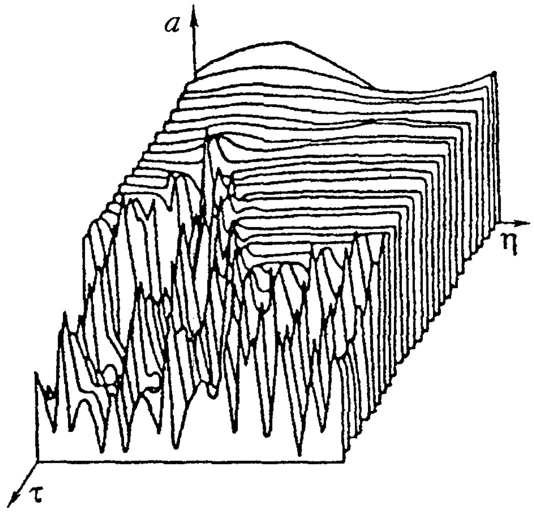
Let us give some examples that characterize the occurrence of self-organization and turbulence.

Figure 1 presents the development of a random field of amplitude and phase perturbations in the medium without dispersion ($\alpha_{11} = \alpha_{12} = \alpha_2 = 0$) that leads to self-organization. At the initial instant ($\tau = 0$), the random field of amplitude and phase perturbations is chosen from the table of random numbers. In the course of development, condition 1 is used. The characteristics of the wave packet, represented by formula (3), evolved to a monochromatic envelope wave obtained as a result of the nonlinear interaction of perturbations (condition 3). In this case, the continuous spectrum of the wave numbers of the wave packet is contracted. The wave packet evolves to a monochromatic wave (condition 4, Fig. 2).

In the case of the nonlinear dependence of the phase (frequency) on the amplitude, the graph of the dependence of the perturbation amplitude a takes the form of sharp wedges (Fig. 3). With multimode instability the perturbations, belonging to the wide band of the spectrum of wave numbers, are excited and grow. As the multimode turbulence develops, expansion of the wave packet occurs (Fig. 4). The amplitudes symmetric relative to the center of the wave packet are not equal to each other. The perturbation energy is rather uniformly distributed over the spectrum of the excited wave packet.

The trajectories of the initially close systems diverge exponentially. The multimode turbulence develops in the system.

From Figs. 3 and 4, it follows that a monoharmonic wave was imposed initially during the process (natural fluctuation interaction) or upon the process (forced interaction). Then, by means of particular interactions, forced or natural ones, condition 2 was satisfied, namely, the phase or the frequency was nonlinearly



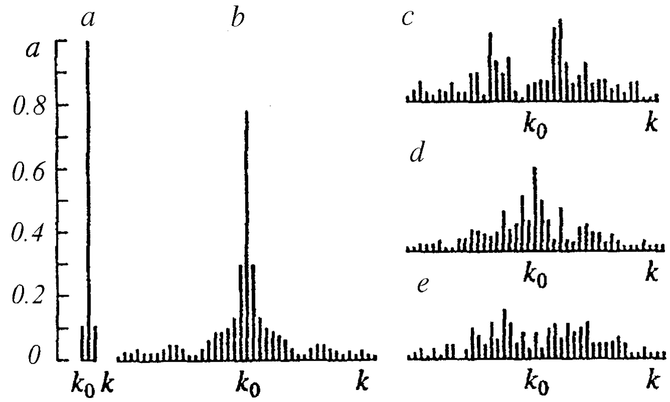


Fig. 3. Evolution of the envelope wave in the case of occurrence of turbulence.

Fig. 4. Expansion of the wave packet when multimode instability develops in the medium with $\alpha \neq 0$ for τ = 0 (a), 5 (b), 10 (c), 15 (d), and 20 (e).

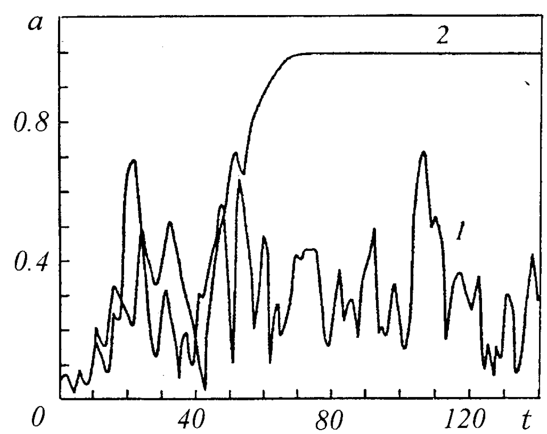


Fig. 5. Development of the carrier-wave amplitude in the medium for $\alpha_1 = 0$; $\alpha_2 = 3$; $\alpha_3 = 0$. t, sec.

dependent on the amplitude. In this case, according to Fig. 4, the spectrum of the wave numbers was expanded. The transition to turbulence occurred, as is evidenced by all the well-known criteria for dynamic systems.

Thus, condition 4 is obligatory for all transitions that characterize the qualitative changes in the trajectory of dynamic systems, i.e., the transition of the system to self-organization is accompanied by the narrowing of the spectrum, while the transition to turbulence, by the expansion of the spectrum. In both cases, the transition is accompanied by energy transfer over the spectrum (Figs. 2, 4).

The occurrence of multimode turbulence can be prevented by increasing the linear dispersion or decreasing the nonlinear dependence of the frequency on the amplitude. The increase in the dispersion favors the formation of coherence-type structures. In flows of an incompressible fluid, the increase in the dispersion of Tollmien–Schlichting waves can be obtained by changing the composition of a moving medium and by introducing high-molecular-weight compounds into the fluid. The behavior of the nonlinear interaction of the waves can also be changed by decreasing the period length L or increasing considerably the amplitude of the initial monochromatic perturbation. The numerical calculations of Eq. (18) indicated that in a linearly no-dis-

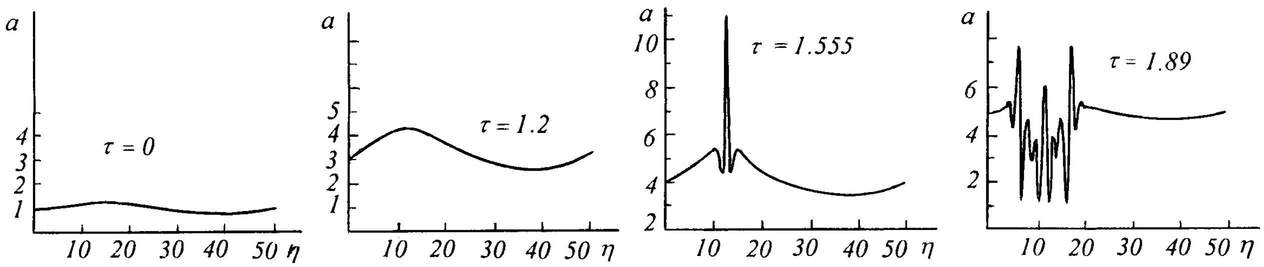


Fig. 6. Stage of the development of a turbulent spot from a needle-shaped wave packet in the case of modulation instability.

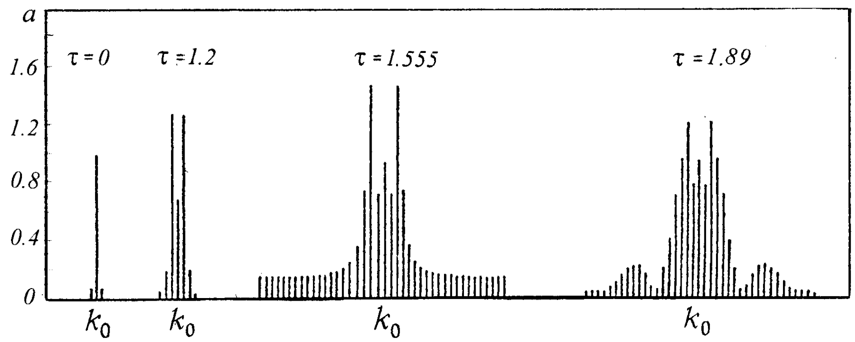


Fig. 7. Evolution of the spectrum of wave numbers during the formation of a needle-shaped wave packet and a turbulent spot in the medium with linear attenuation.

persion medium with a strong dependence of the phase on the amplitude the action of white noise on the multimode turbulent regime can lead to the establishment of an ordered monochromatic regime or a limiting cycle in the system. Figure 5 shows the evolution of the carrier-wave amplitude in the system with $\alpha_{11} = \alpha_{31} = 0$, $\alpha_2 = 3$, and $L = 50$ in the absence of noise (curve 1) and in the presence of white noise (curve 2). The self-organization of chaotic regimes under the action of white noise is explained by the fact that in non-dispersion systems together with limiting cycles and chaotic attractors there are stable nodes that correspond to monochromatic regimes. White noise favors the system getting into the domain of attraction of a stable node.

As a result of the dispersion spreading of a wave packet, the propagation velocity of the fronts of a localized wave packet in the dispersion media is higher than in nondispersion ones. With increase in the wave-packet amplitude, the phase changes, owing to which the difference in the local wave numbers on the wave-packet fronts from the wave number of the carrier wave grows, thus causing the self-constriction of the wave packet similarly to that in conservative systems. The self-constriction that is manifested in a sharp increase in the amplitude in the vicinity of the crest of the envelope wave leads to the appearance of narrow needle-shaped wave packets of large amplitude. The large amplitude gradients on the fronts of the needle-shaped wave packets cause the growth in the phase gradients resulting in the appearance of local decrements and the attenuation of these packets. The evolution of the envelope wave in the systems for $\alpha_{11}\alpha_2 < 0$ is determined by the dynamics of the appearance, growth, and attenuation of the needle-shaped wave packets in which the main portion of the wave-motion energy is concentrated. In the systems with $\alpha_{11}\alpha_2 < 0$ the monochromatic exponentially increasing solution is unstable. The development of instability is accompanied by the appearance of separate peaks of large amplitude. Subsequently, in failure of these peaks turbulent spots are formed that propagate to the entire wave packet. The individual stages of formation of a needle-shaped wave packet, leading to the occurrence of a turbulent spot, and also the energy transfer over the spectrum are given in Figs. 6 and 7. It should be noted that the occurrence of the turbulent spot is similar to the well-known

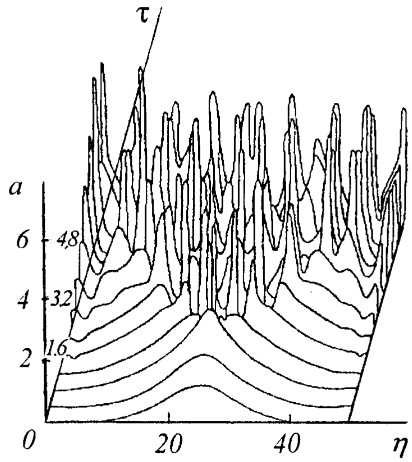


Fig. 8. Self-constriction of the wave packet that leads to the appearance of needle-shaped wave packets, i.e., a system of turbulent spots. $\alpha_1 = 4$, $\beta_1 = 0$.

phenomenon, i.e., to the θ layer [4]. The difference lies in the fact that the occurrence of turbulent spots (as follows from Figs. 6 and 7) is characteristic of both animate and inanimate nature. The pattern of development of several turbulent spots is shown in Fig. 8.

Classification of supercritical regimes is one of the central problems of the theory of dissipative structures [16]. It allows one to separate individual substantial aspects of an object or phenomenon and to determine the conditions for transition between them.

The supercritical regimes, in which on many scales multiple and nonmultiple harmonics can considerably affect the behavior pattern of the nonlinear systems on the attractor, are classified according to the type of envelope wave, the spatial spectra of wave numbers, and the time frequency spectra of the amplitudes and phases of individual harmonics [10].

Mechanism of Occurrence of Turbulence. As has been noted above, the envelope wave is a topological mapping of the nonlinear interaction of perturbations. For the multimode turbulence the envelope wave is broken into wedge-shaped wave packets with rather large spatial and frequency spectra. The inhomogeneity in group velocity appears in the medium. The group velocity of short waves entering into the wave packet is higher than that of the carrier wave itself, as a result of which the short waves overtake the carrier wave and are concentrated on the leading edge of the wave packet. The group velocity of long waves is lower than that of the carrier wave; therefore, the long waves are at the tail end of the wave packet. The nonlinear long-term development of perturbations is investigated by the method of wave packets in the supercritical region for the continuous band of the spectrum of wave numbers, and it is shown that waves of various nature are formed on different sides of the carrier wave. Owing to the nonlinear interaction, each type of waves forms attractors, including "strange" ones. We observe the interaction between the "strange" attractors. Natural perturbations are wave packets [17] containing tens of modes, while the number of efficiently interacting modes that appear at combined wave numbers in excitation of two modes is equal (for the nonlinear dependence of the phase (frequency)) to tens and hundreds of modes. The number of modes increases in geometric progression; therefore, the filling of the spectrum of wave numbers occurs very rapidly. Thus, turbulence is a result of the interaction of a rather large number of "strange" attractors. This conclusion agrees with the previously stated assumption of the mechanism of occurrence of turbulence [18, 19] and with the results of experimental investigations of the turbulence in a round tube [20]. The mechanism proposed does not contradict the existing models according to which large-scale perturbations (long waves at the tail end of the wave packet) turn out to be unstable and produce perturbations with a small scale (short waves on the leading edge of the wave packet). Since the concentration of waves at the wave-packet boundaries is something material, the above fact is also in agreement with the conclusions given in [18, 19] that in the case of the occurrence of turbulence the "quasiparticles" composing the "gas" of turbulence are of considerable importance.

Thus, based on the analysis of the numerical solutions of a general nonlinear parabolic equation, we have established the laws of occurrence of self-organization and turbulence. The conditions for the appearance of turbulent spots and for the cancellation of turbulence by "white noise" have been shown. The mechanism of occurrence of turbulence has been suggested.

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